Planes

What is the definition of a plane?

A **plane** is a flat surface that extends in all directions. Can be defined by 3 non-colinear (not on a line) points or by a point and a vector (which is perpendicular to the plane).

What is the equation for a plane?

The general equation for a plane is:

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Ax + By + Cz + D = 0
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An alternate form of this equation is:

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A'x + B'y + C'z + D' = 0

A'= A/d, B'= B/d, C'= C/d, D'= D/d
where d = sqrt(A^2 + B^2 + C^2)
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How can I find the normal vector to a plane?

Given three points (P1, P2, P3) the normal vector N is given by:

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N = [P2 - P1] X [P3 - P1]
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where the operation "X" is the cross product.

Given the equation for a plane:

```
Ax + By + Cz + D = 0
```

the vector [A B C] is normal to the plane.

How can I find the equation for a plane?

Given three points: P1, P2, P3

- Find the normal vector, N = [A B C].
- Find the coefficient D, by taking the dot product of N with any point P in the plane and negating the result:

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N . P = -D
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- Use the coefficients found (A,B,C,D) in the general equation for a plane.

How can I find the intersection of two planes?

The intersection of two non-parallel planes is a line. The equation for this line can be found by solving the simultaneous equations of the planes. This can be done using a general method such as Gaussian elimination.

How can I find the intersection of a plane and a line?

The intersection of a plane and a line that are not parallel is a point. This point can be found by solving the simultaneous equations for the plane and the line using a general method such as Gaussian elimination.

How can I determine if a given point is in a particular plane?

Given a point P = (x,y,z), and a plane with normal N and point P1:

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N . [P - P1] = 0
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if P is in the plane.

How can I determine if a given line is in a particular plane?

Given two points on the line, P1 and P2:

> If both points are in the plane, the line is in the plane.

Given a line defined by vector V and point P:

> If P is in the plane and V is perpendicular to N (the normal to the plane), the line is in the plane.

What is a half-plane?

A line which lies in a plane divides that plane into two **half-planes**. A half-plane expresses an inequality. For example, for the equation $x + y > 2$, all the points in the x-y plane that satisfy the inequality are in one half-plane and those that do not are in the other. The dividing line is the equation $x + y = 2$.